X-643-69-127
PREPRINT

NASA TM X-63509

THERMAL AND TIDAL EFFECT ON THE LIBRATION OF MERCURY

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MARCH 1969

GSFC

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ABSTRACT

It is shown that the influences of the thermal and tidal effects on Mercury's libration are in an equilibrium condition with the periods of rotation and revolution of Mercury locked in the 3:2 resonant state.

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INTRODUCTION

Liu (Liu, 1968a; Liu, 1968b) has shown that the trapping of Mercury's rotational period into a 3:2 resonance lock with its orbital period was originally affected by the thermal contraction of the figure of Mercury during solidification. In the present paper the analysis of the contribution of the two thermal bulges to the dynamic stabilization of the planet's libration is given. Attention is focused on the balance of the influence of the tidal and thermal effect on the libration of Mercury after solidification.

BASIC EQUATIONS

After solidification, the thermal contraction of the figure of Mercury from loss of heat must be exceedingly small and the thermal bulges considered by Liu (Liu, 1968a; Liu 1968b) can grow because the apparent circulational motion at successive perihelia has been converted to a librational motion. The variation with time of the fractional difference $\left[B_{(t)} - A_{(t)}\right]/C$ in Mercury's equatorial moments of inertia is

$$\frac{B_{(t)} - A_{(t)}}{C} = \lambda + \frac{3\left(2 - 5\beta^3 + 3\beta^5\right)}{100\left(1 + \beta + \beta^2\right)} \cdot \alpha \cdot \Delta T \tag{1}$$

in which α is the coefficient of linear thermal expansion, ΔT the difference in surface temperature between the regions around the perihelion and the aphelion

axes of Mercury, and $\beta = \left(R_m - y\right)/R_m$ where R_m is Mercury's radius and y the depth of solar heating. The value of λ has been estimated as $\lambda \geq 5 \times 10^{-5}$ and the rate of increasing of $\left[B_{(t)} - A_{(t)}\right]/C$ is about 10^{-21} sec.$^{-1}$ (Liu and O'Keefe, 1965; Liu, 1968a; Liu, 1968b).

In considering the bodily tidal torque we have recourse to the estimation by Jeffreys. (Jeffreys, 1959).

$$N = \frac{1}{50} \cdot \frac{18}{5} \pi G \rho \frac{M_s^2}{M_m^2} \cdot \frac{R_m^6 \left(1 + e \cos f\right)^6}{a^6 \left(1 - e^2\right)^6} \cdot C \cdot \mathrm{Sin}(2\epsilon) \tag{2}$$

where G is the gravitational constant, ρ density, M_s mass of the Sun, M_m mass of Mercury, f the true anomaly, a the semimajor axis, e the orbital eccentricity and ϵ the phase lag of the conventional equilibrium tides.

The orientation of Mercury relative to the Sun, ϕ, is then governed by (Liu and O'Keefe, 1965)

$$\frac{d^2 \varphi}{df^2} - \frac{2e \,\mathrm{Sin}\, f}{1 + e \cos f} \left(\frac{d\varphi}{df} + 1\right) + \frac{3(\lambda + \Delta\lambda)}{2(1 + e \cos f)} \cdot \mathrm{Sin}\, 2\varphi = -\frac{N}{n^2 C} \cdot \frac{\left(1 - e^2\right)^3}{(1 + e \cos f)^4} \tag{3}$$

in which

$$\Delta\lambda = \frac{3\left(2 - 5\beta^3 + 3\beta^5\right)}{100\left(1 + \beta + \beta^2\right)} \cdot \alpha \cdot \Delta T$$

and n is the mean orbital angular velocity.

ANALYSIS

Let us choose the time τ reckoned from perihelion and related to the orbital period divided by 2π

$$\tau_{(f)} = 2 \tan^{-1} \left(\frac{1-e}{1+e}\right)^{1/2} \tan \frac{f}{2} - \frac{e(1-e^2)^{1/2} \operatorname{Sin} f}{1+e \cos f} \tag{4}$$

As a new unknown function, we take the angle of rotation Ψ between the axis of the two thermal bulges and the radius vector of the perihelion

$$\Psi = f + \phi \tag{5}$$

Equation (3) takes the form

$$\frac{d^2 \Psi}{d\tau^2} + \frac{3(\lambda + \Delta\lambda)}{2} \cdot \frac{(1 + e \cos f)^3}{(1 - e^2)^3} \operatorname{Sin} 2(\Psi - f) = - \frac{N}{n^2 C} \tag{6}$$

The solution of Equation (6) may be sought in the form.

$$\Psi = \Omega\tau + \delta$$

where Ω is a constant and δ is an unknown function. The resonances occur at $2\Omega = m$ if m is an integer and δ is, then, the angle of libration. To obtain an approximate solution we average it over a period of 2π. For $m = 3$, Equation (6) becomes

$$\frac{d^2 \delta}{d\tau^2} + \frac{2 \cdot 1}{2} (\lambda + \Delta\lambda) \operatorname{Sin}(2\delta) = - 10^{-9} \operatorname{Sin}(2\epsilon) \tag{7}$$

In the derivation of Equation (7), the following values were adopted:

$$e = 0.2$$

$$G = 6.7 \times 10^{-8} \text{ dyn. } Cm^2 \cdot g^{-2}$$

$$\rho = 5.0 \text{ g} \cdot Cm^{-3}$$

$$n = 1.2 \times 10^{-6} \text{ rad.} \cdot Sec^{-1}$$

$$M_s/M_m = 6.0 \times 10^6$$

$$R_m/a = 4.1 \times 10^{-5}$$

The average process also included the effect of the sign of the phase lag which depends on the sign of $d\varphi/d\tau$. In Equation (7) the value of $\lambda + \Delta\lambda$ is about 5×10^{-5} and the tidal torque is much less than the maximum restoring thermal torque. Therefore, the rotation of Mercury at the 3:2 resonance state is seen to be stable.

The first integral of Equation (7) is

$$\frac{1}{2}\left(\frac{d\delta}{d\tau}\right)^2 - \frac{2.1}{4}(\lambda + \Delta\lambda)\ \mathrm{Cos}(2\delta) = -10^{-9}\ \mathrm{Sin}(2\epsilon)\cdot\delta + E_0 \qquad (8)$$

where E_0 is a constant.

Differentiating Equation (8) with respect to t, we obtain

$$\frac{1}{2}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 - \frac{2\cdot 1n^2}{4}(\lambda + \Delta\lambda)\frac{d\ \mathrm{Cos}(2\delta)}{dt}$$

$$= \frac{2\cdot 1n^2}{4}\cdot\frac{d(\Delta\lambda)}{dt}\cdot\mathrm{Cos}(2\delta) - 10^{-9}\,n^2\,\mathrm{Sin}(2\epsilon)\frac{d\delta}{dt} \qquad (9)$$

Hence,

$$\frac{dE}{dt} = \frac{dE\ (\text{thermal effect})}{dt} - \frac{dE\ (\text{tidal effect})}{dt} \tag{10}$$

in which

$$\frac{dE\ (\text{thermal effect})}{dt} = \frac{2 \cdot 1}{4}\, n^2 \cdot C \cdot \operatorname{Cos}(2\delta) \cdot \frac{d(\Delta\lambda)}{dt} \tag{11}$$

and

$$-\frac{dE\ (\text{tidal effect})}{dt} = -\ 10^{-9}\ n^2 \cdot C \cdot \operatorname{Sin}(2\epsilon)\, \frac{d\delta}{dt} \tag{12}$$

For small librational angle δ and $d(\Delta\lambda)/dt = 10^{-21}\ \text{Sec}^{-1}$, the result of Equation (11) is

$$\frac{dE\ (\text{thermal effect})}{dt} = 0\ (10^{10}\ \text{erg.} \cdot \text{Sec}^{-1})$$

For $\operatorname{Sin}(2\epsilon) \cong 5 \times 10^{-3}$ and $d\delta/dt \leq 10^{-3}\, n$, the result of Equation (12) is

$$-\frac{dE\ (\text{tidal effect})}{dt} = -\ 0\ (10^{10}\ \text{erg.} \cdot \text{Sec}^{-1})$$

It is seen that the thermal expansion effect works against the tidal effect on Mercury's libration at a rate of 10^{10} erg. Sec^{-1}. This is of the same order of magnitude as the rate of the librational dissipation for the bodily tidal friction of Mercury.

CONCLUDING REMARKS

We have shown that the influences of the thermal and tidal effect on Mercury's libration are in an equilibrium condition with the periods of rotation and revolution of Mercury locked in the 3:2 state. With regard to the interaction of the thermal effect, tidal friction and gravitation, we conclude that the growth rate of the thermal bulges on Mercury's surface contributes dynamically to the stabilization of the planet's libration.

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